



Simple features of digital image

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Histogram modelling

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Segemntation

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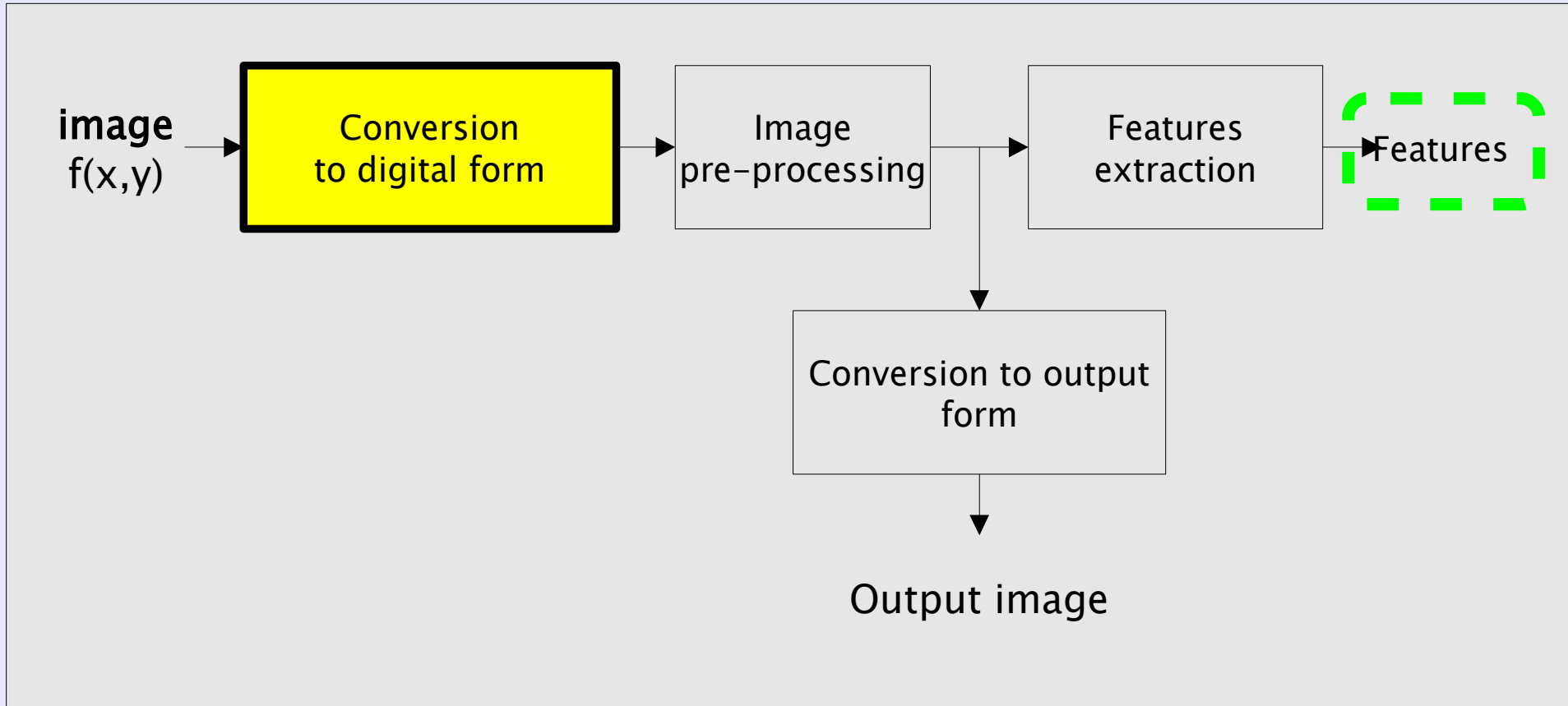


1. luminance and contrast

2. intensity histogram

3. histogram modifications

4. cooccurrence matrix





The influence of luminance and contrast on image's characteristics

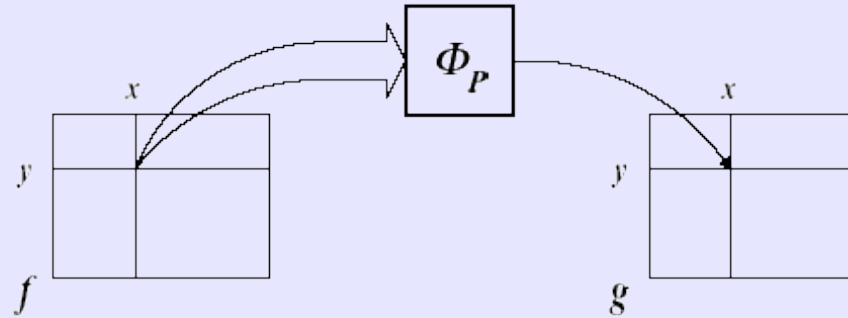


$$J = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N f(i, j)$$

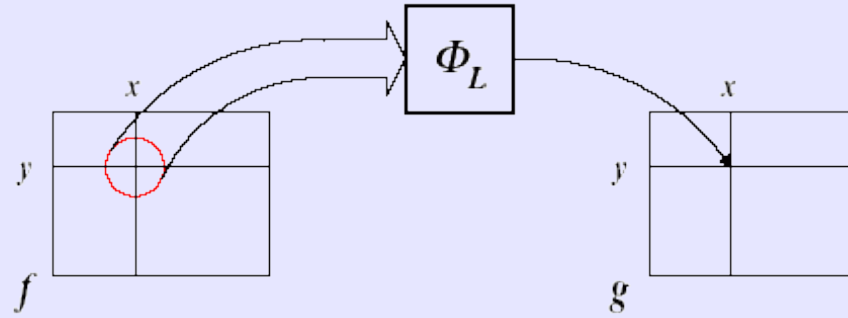
$$C = \sqrt{\frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N [f(i, j) - J]^2}$$



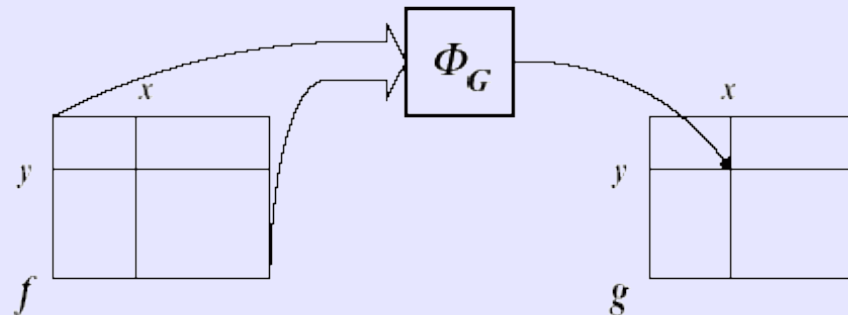
point transform

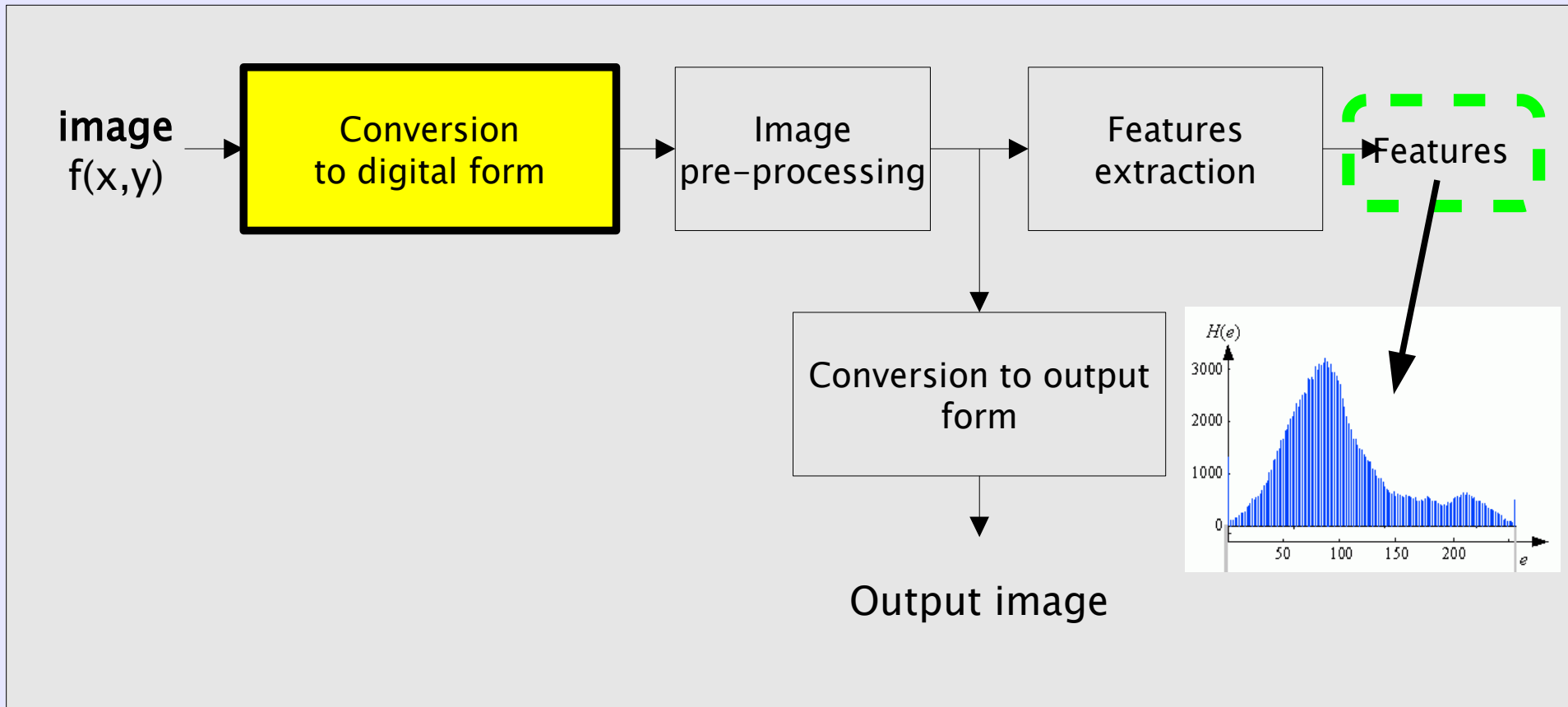


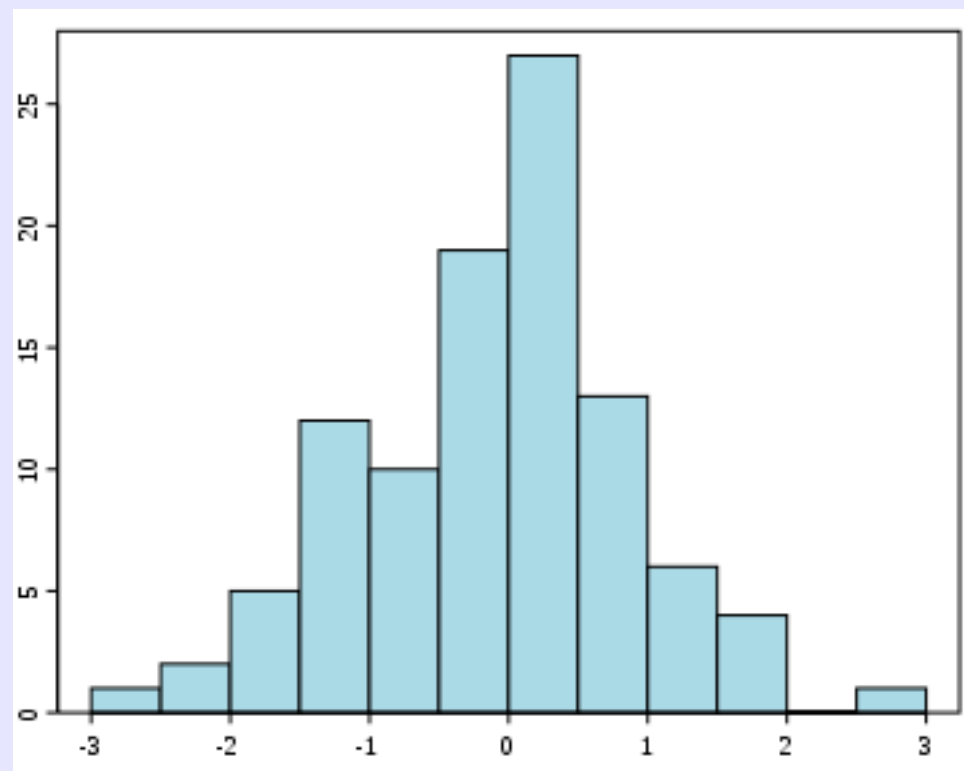
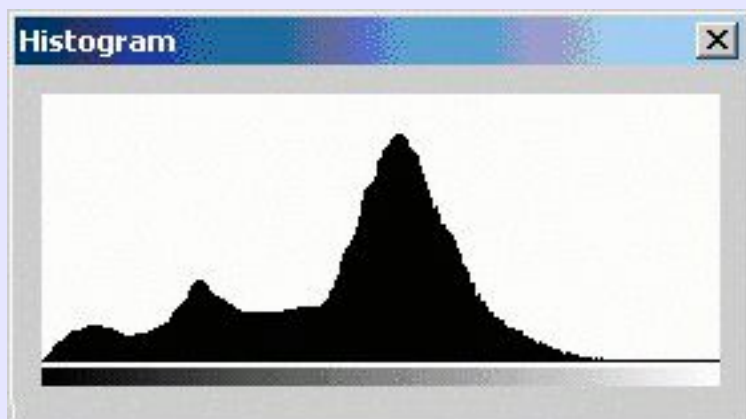
local transform



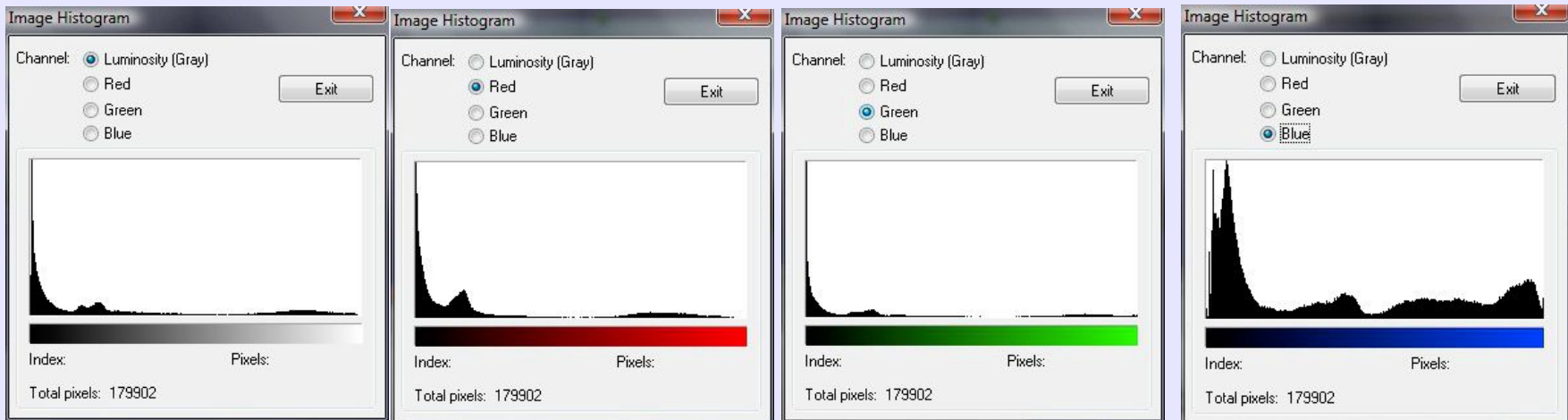
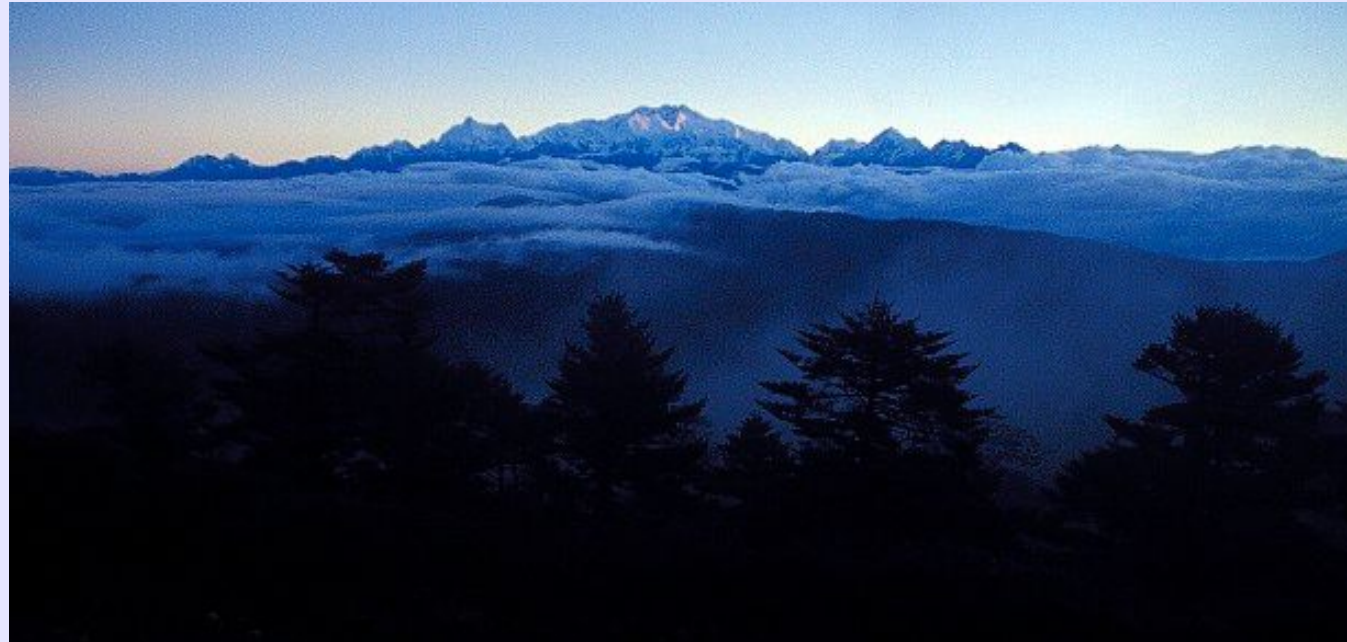
global transform





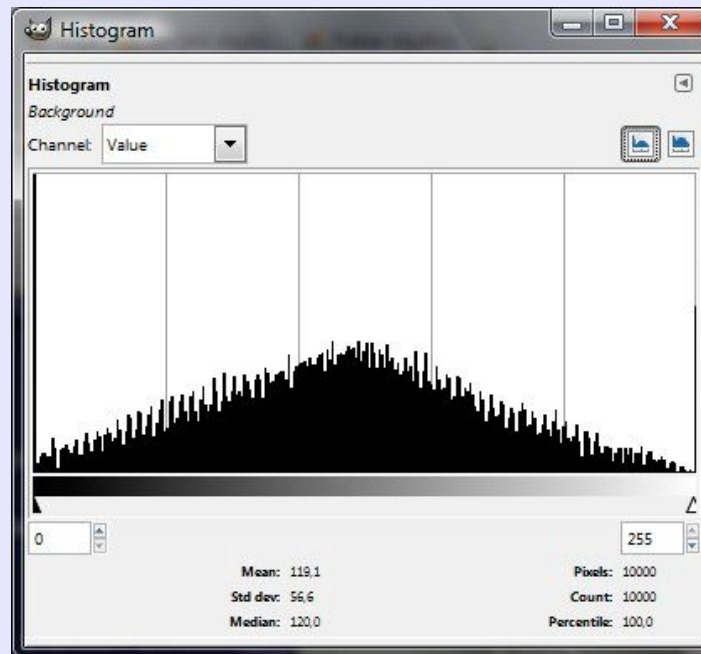
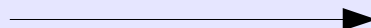
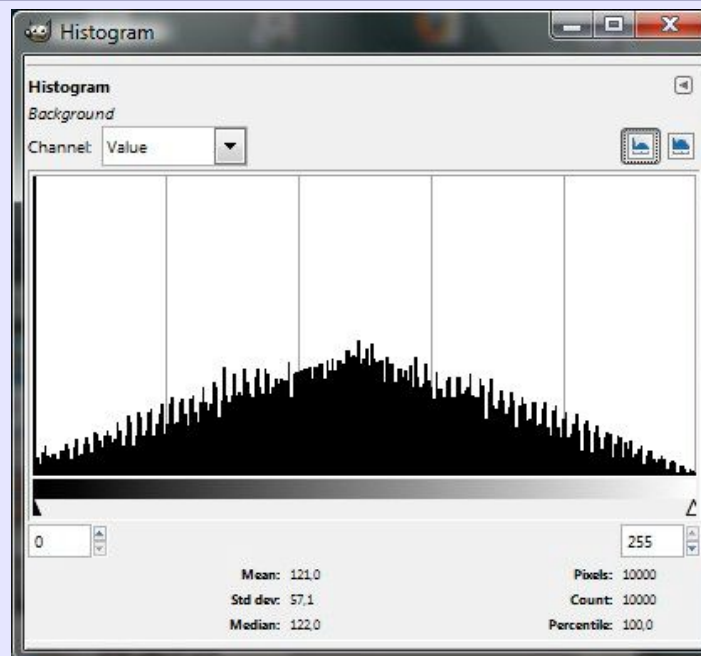


- The most simple case - *one-dimensional histogram*
- Representing the frequency of elements in some set;
 - in case of an image – the number of pixels with adequate intensity level



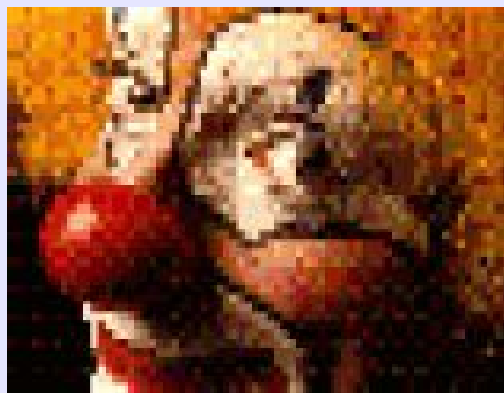
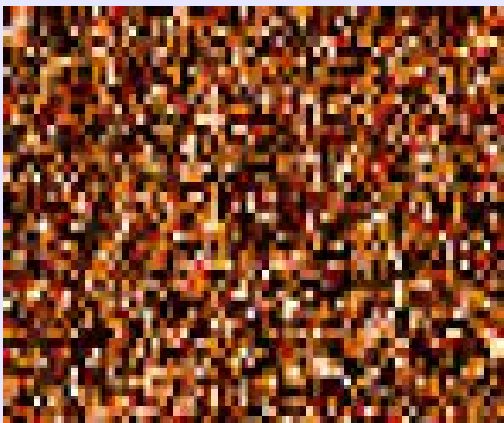


Histogram is not unique!





Examples of equal histograms



From the purely statistical point of view, it is an estimate of the probability distribution of a continuous variable (first introduced by Karl Pearson)

The area under a histogram $H(b)$. (cumulative histogram):

$$\int_0^b H(b) db$$

where:

H(b) – histogram function,

db - step,

H(b)db – height of each histogram interval,

For digital images (db is limited), the histogram is calculated as a sum of all pixels with a certain value, often normalized to 1 (by dividing by the sum of all pixels)



The algorithm of calculating simple intensity histogram for 2D image is as follows:

- Set (or assume) the range of intensity for input image (i.e. 0-1, 0-255, 1-256, etc.);
- Assume the number of intervals;
- Calculate the width of an interval (by dividing range by the number of intervals);
- Count the number of pixels in each interval.

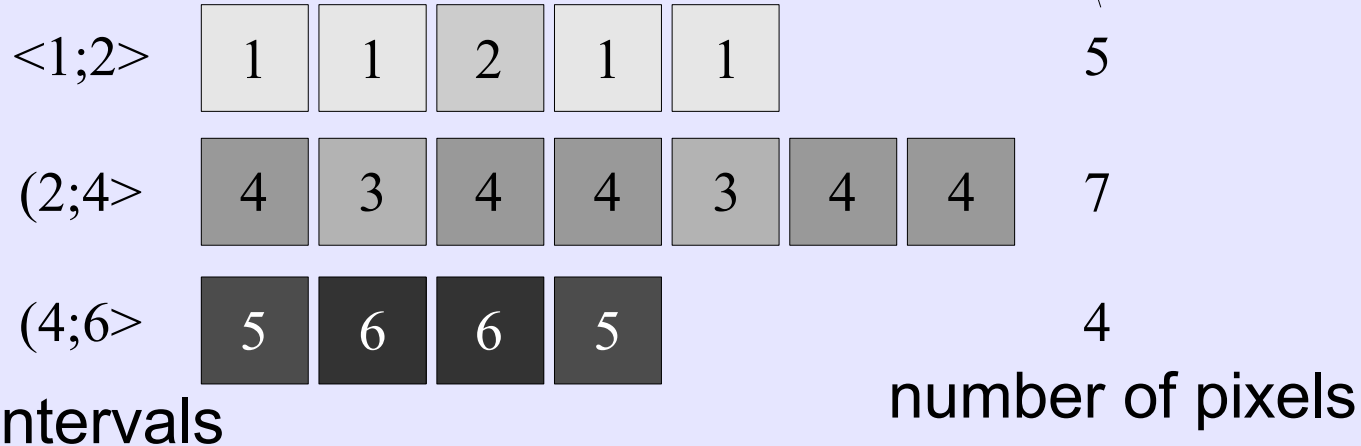
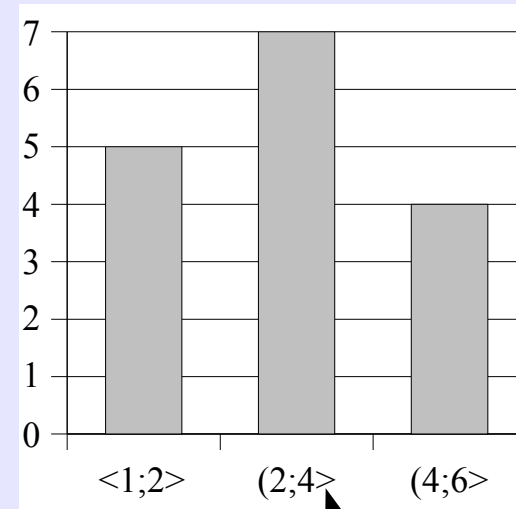


The idea of calculations

image

1	4	1	3
5	6	2	6
4	4	1	3
4	1	5	4

histogram

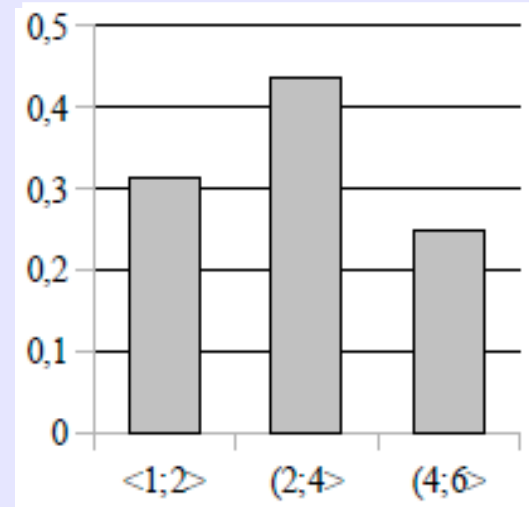




image

1	4	1	3
5	6	2	6
4	4	1	3
4	1	5	4

histogram



<1;2>

1	1	2	1	1
---	---	---	---	---

5 / 16

(2;4>

4	3	4	4	3	4	4
---	---	---	---	---	---	---

7 / 16

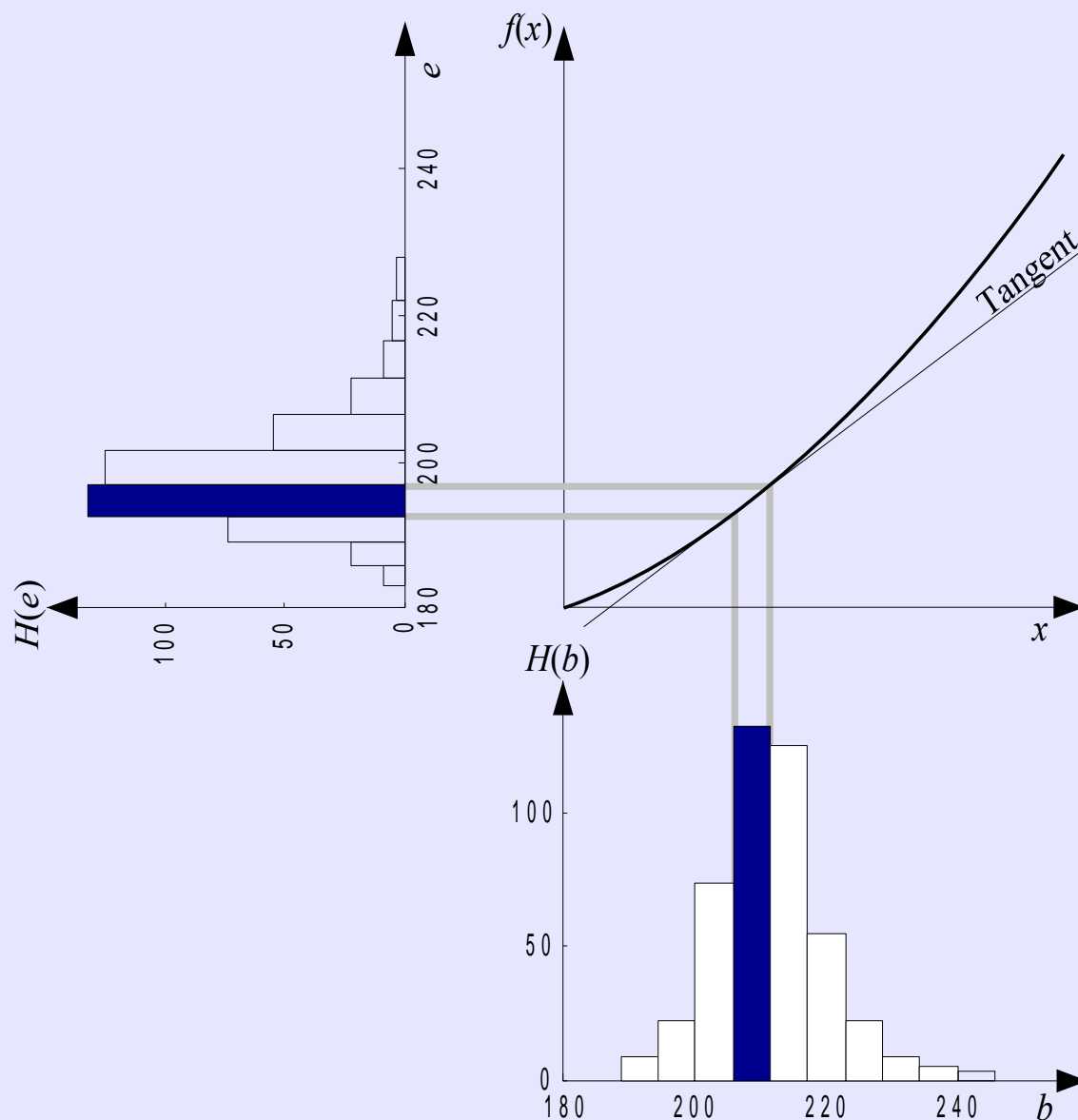
(4;6>

5	6	6	5
---	---	---	---

4 / 16

intervals

normalization



Histogram stretching along a defined line changes the **distribution of intensities** in an image by the alternation of **intensity assignment** in each interval

Each interval changes its width:

$$d e = f'(b) d b$$

where

- b – pixel intensity before;
- e – pixel intensity after stretching;
- $f(b)$ – stretching function.

The tangent of an angle of function $f(b)$ is the coefficient that changes the width of each histogram interval

The most simple is a linear stretching:

$$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ ax & \\ E & \text{for } x > E \end{cases}$$

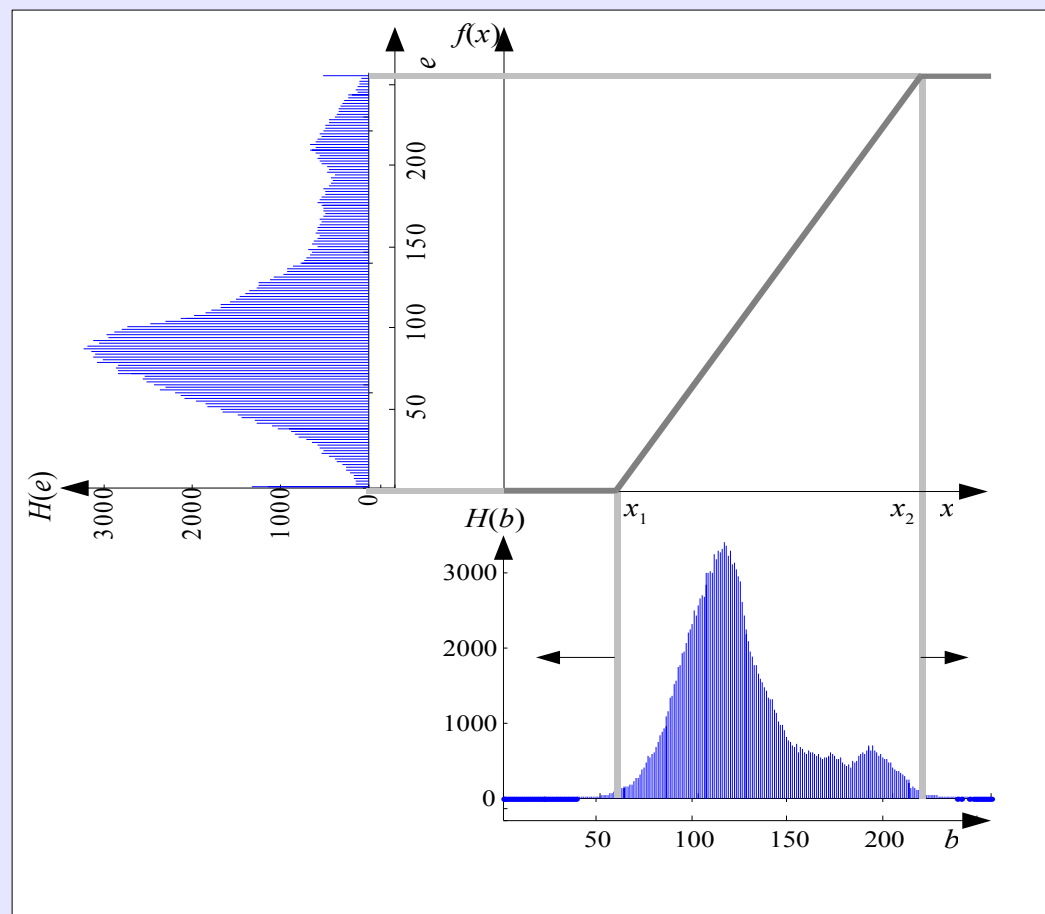
Where a can be equal to:

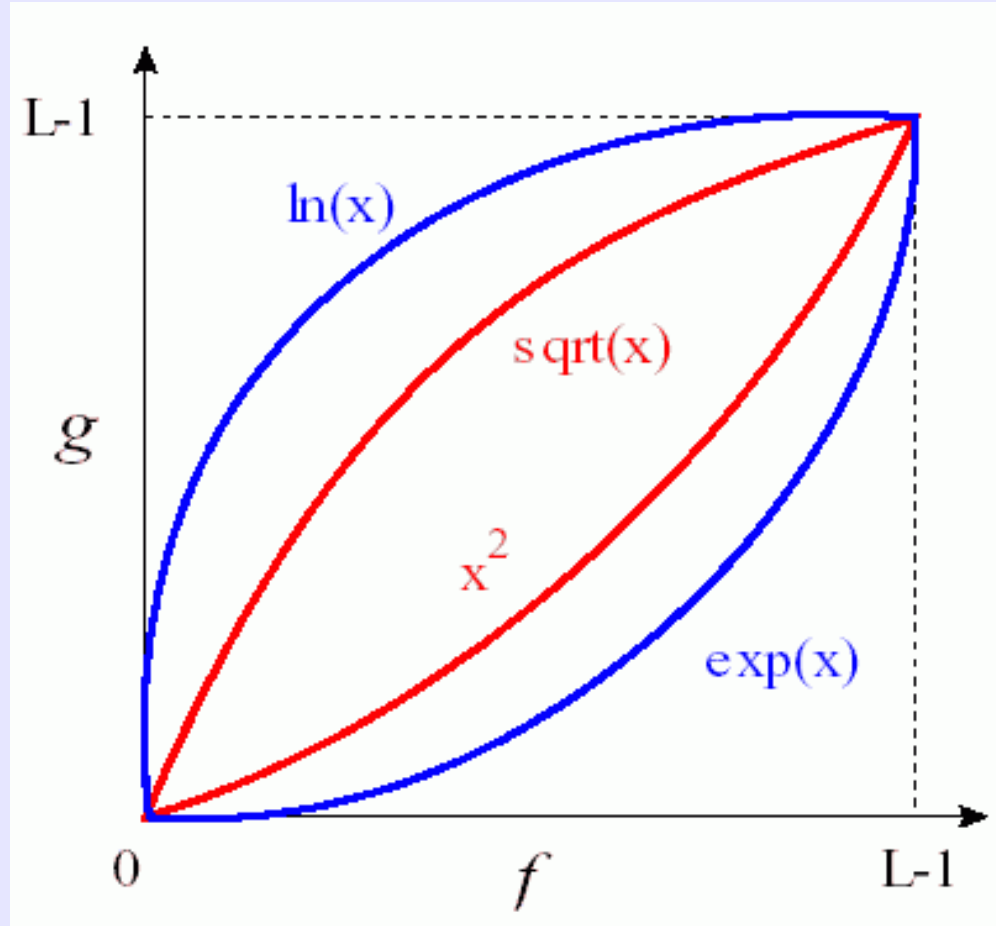
$$a = \frac{E}{x_2 - x_1}$$

where

x_1, x_2 – boundaries of intensity.

E – maximum possible intensity

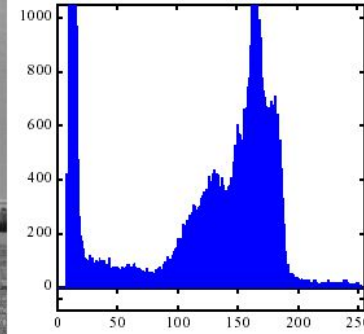






Non-linear cases (examples)

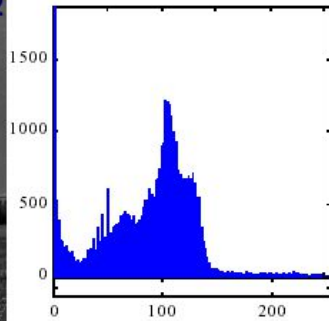
source →



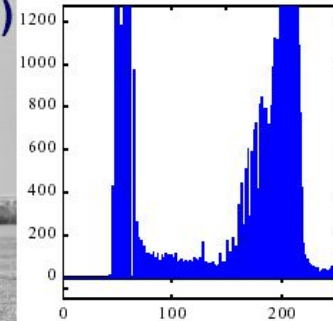
← histogram



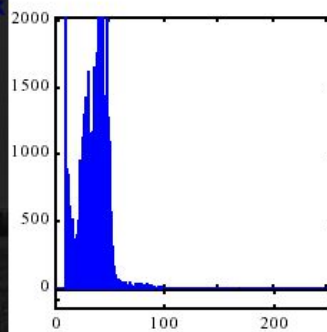
$$T=x^2$$



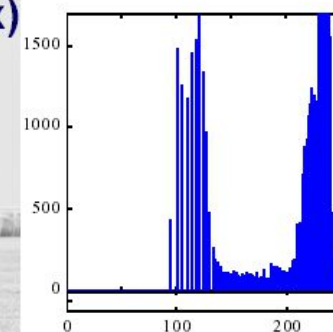
$$T=\sqrt{x}$$



$$T=e^x$$



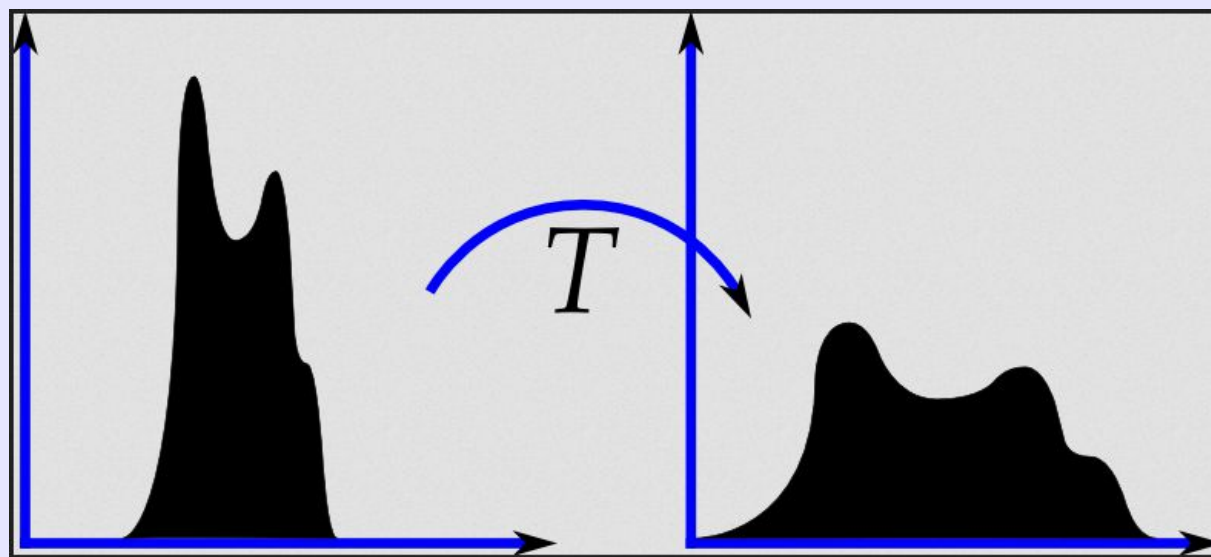
$$T=\log(x)$$



It usually increases the global contrast of images, especially when the usable data of the image is represented by close contrast values.

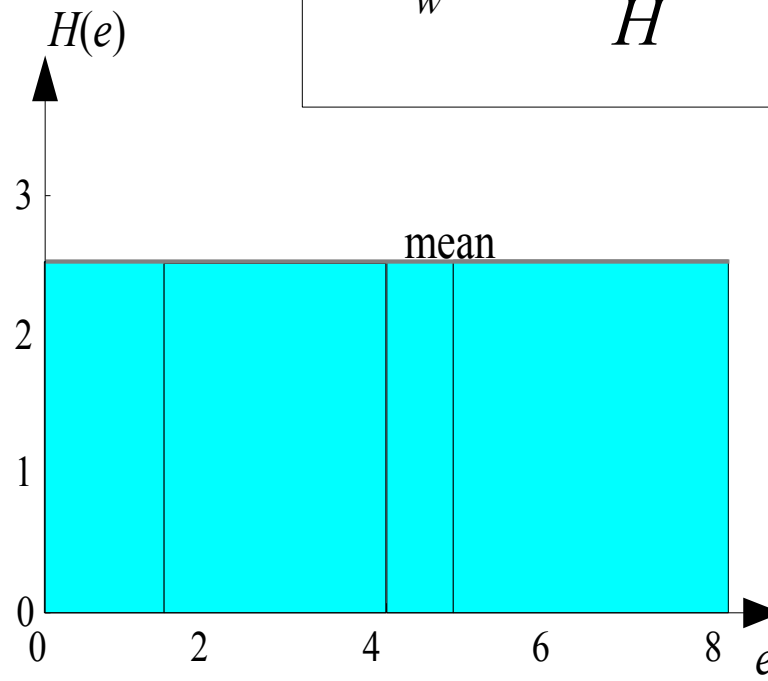
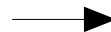
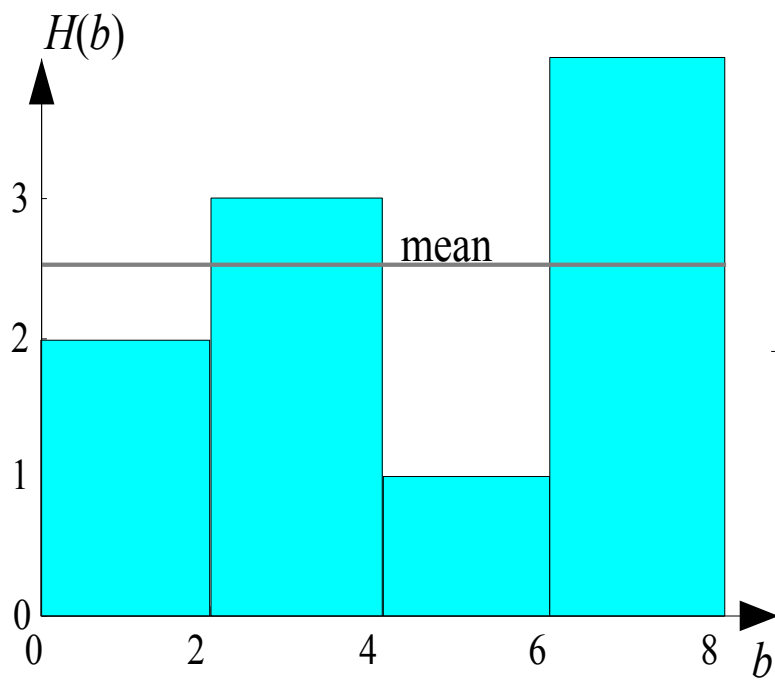
Through this adjustment, the intensities can be better distributed on the histogram.

Areas of lower local contrast gain a higher contrast.



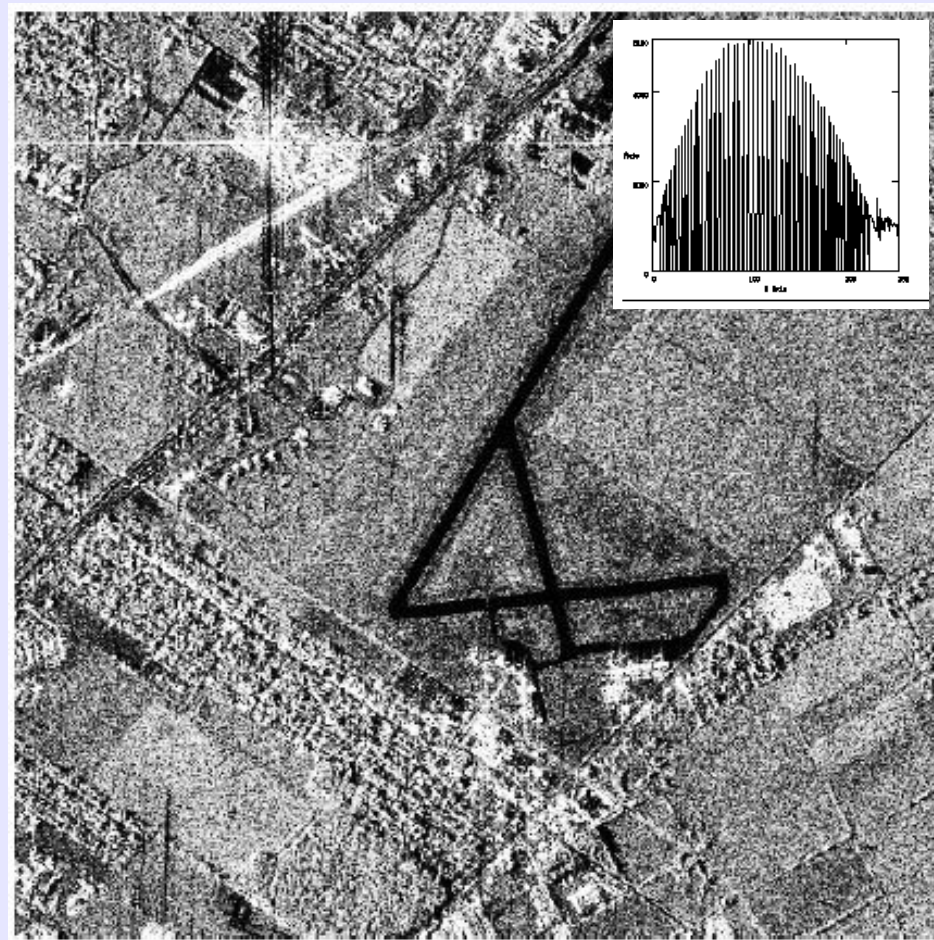
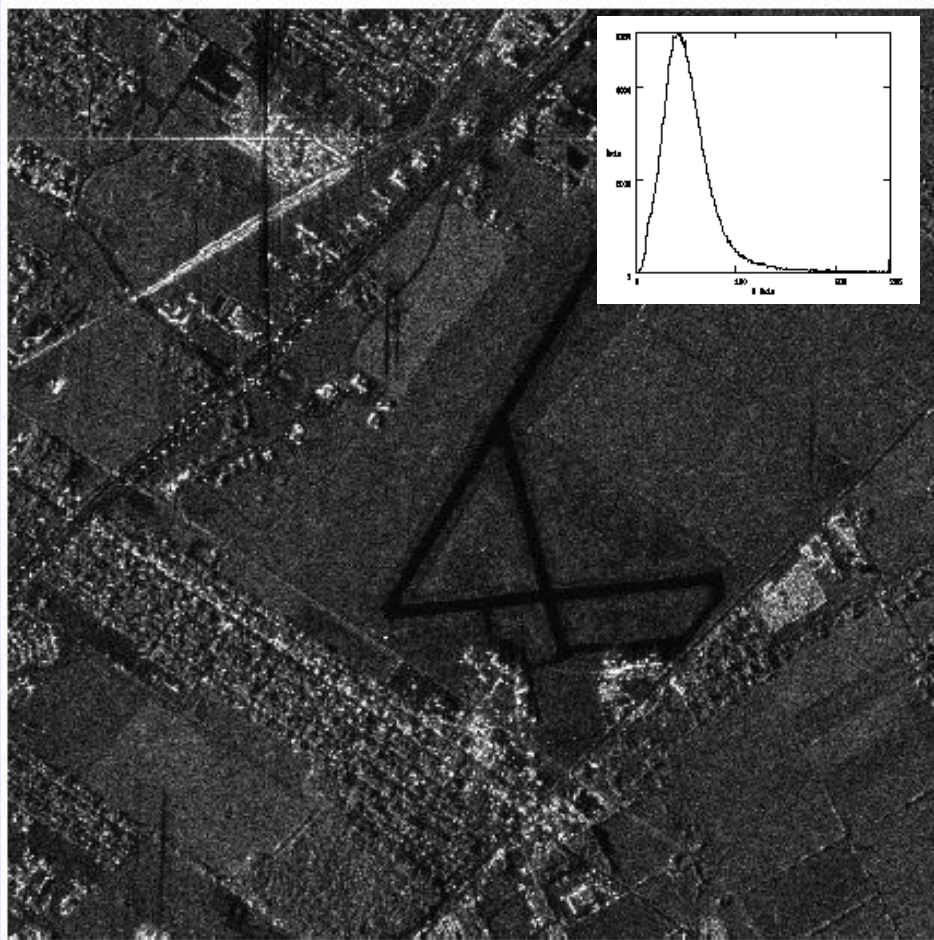
New pixel values can be calculated as follows:

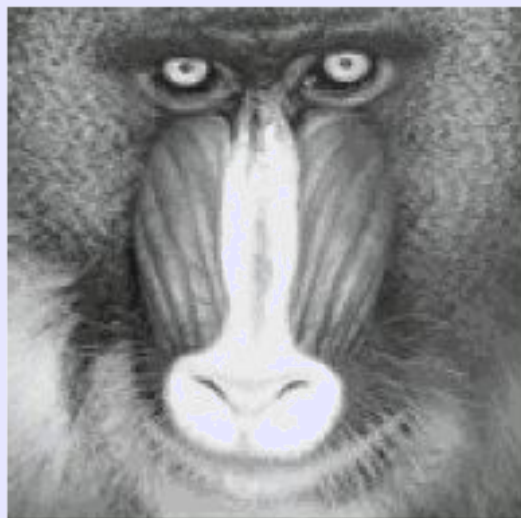
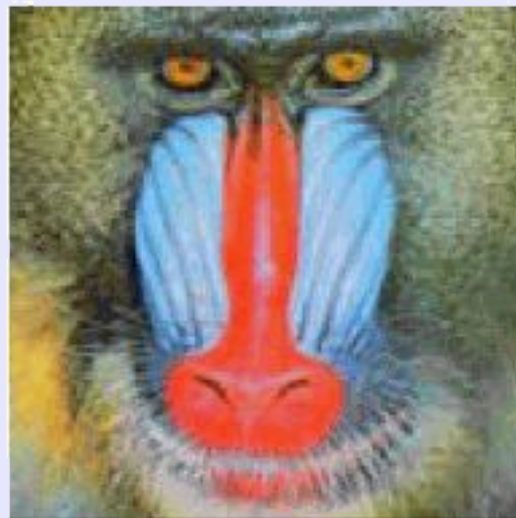
$$o_w = \frac{oH[b(o)]}{\overline{H}}$$



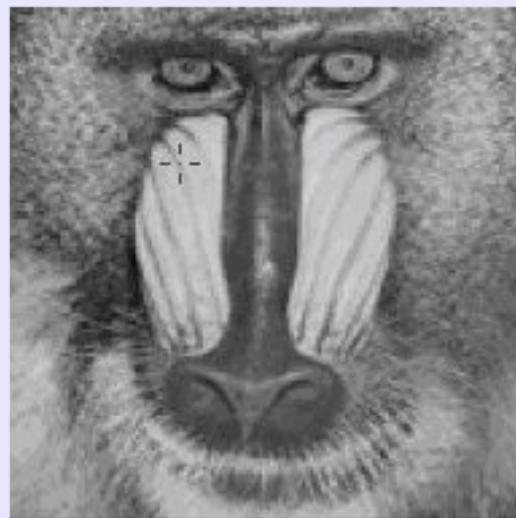
Let us assume an image with 64 x 64 pixels with one of eight intensities: $K=8$, $n=64*64= 4096$

q_k	n_k	$H(q_k) = \frac{n_k}{n}$	$w_k = \sum_{j=0}^k H(q_j)$	$v_k = w_k$
0	790	0.19	0.19	$\frac{1}{7}$
$\frac{1}{7}$	1023	0.25	0.44	$\frac{3}{7}$
$\frac{2}{7}$	850	0.21	0.65	$\frac{5}{7}$
$\frac{3}{7}$	656	0.16	0.81	$\frac{6}{7}$
$\frac{4}{7}$	329	0.08	0.89	$\frac{6}{7}$
$\frac{5}{7}$	245	0.06	0.95	1
$\frac{6}{7}$	122	0.03	0.98	1
1	81	0.02	1.00	1

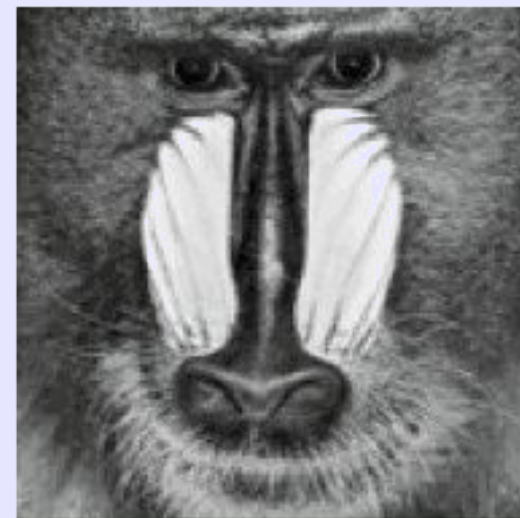




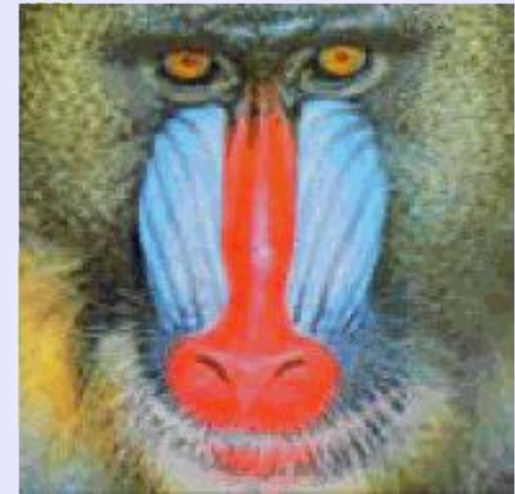
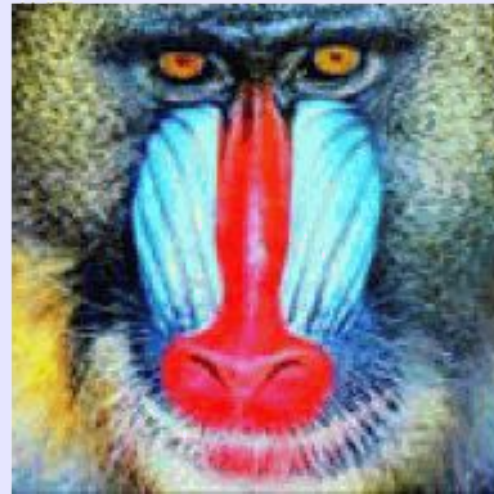
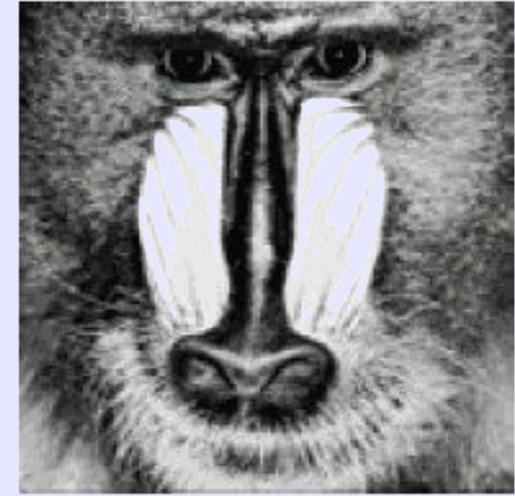
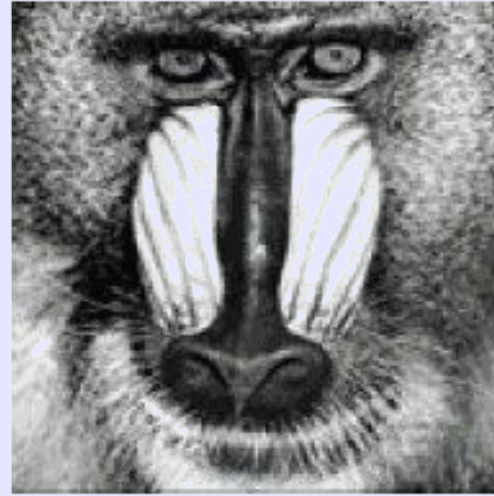
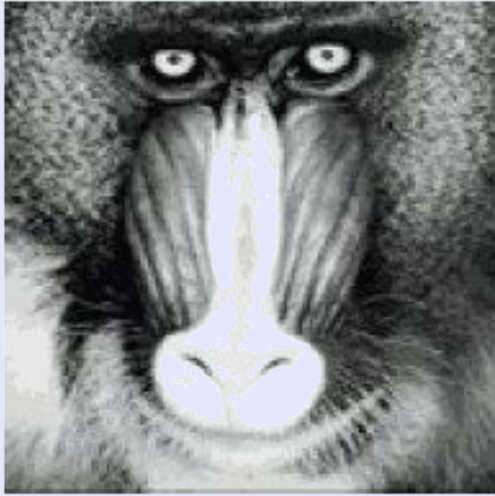
R



G



B



RGB equalized

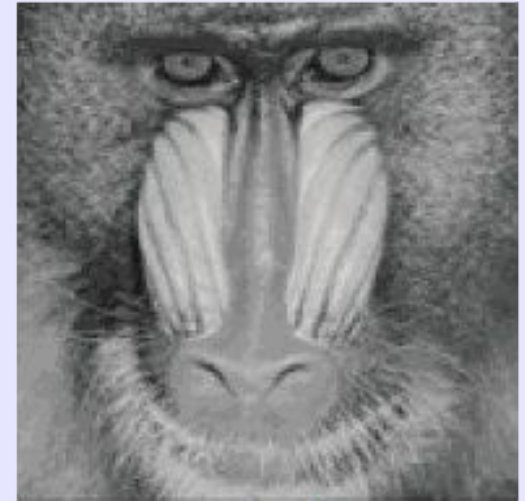
original



H ↓



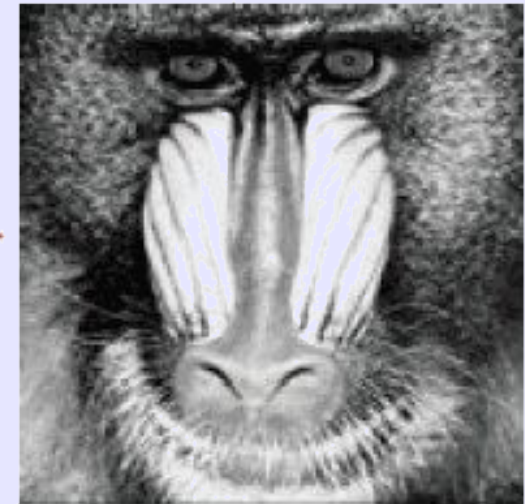
S ↓

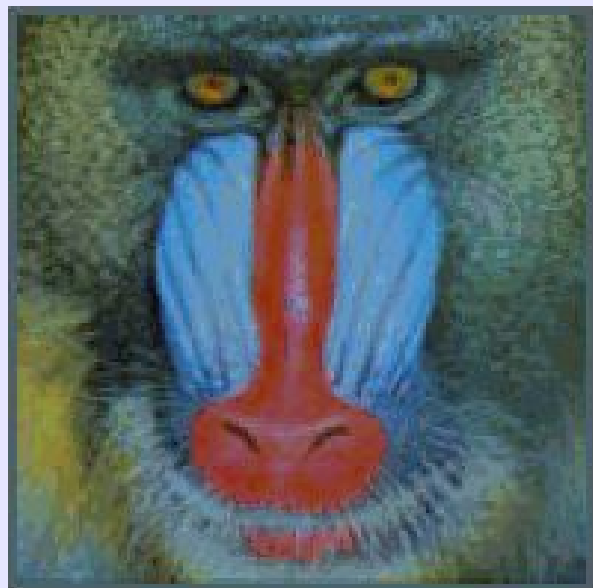


L ↓



HSL equalized





original



RGB equalized



HSL equalized



Another example (RGB vs HSL)



original



RGB



HSL

One-dimensional histogram is defined by function f :

$$\begin{aligned} f &: X \times Y \rightarrow Z \\ f^{-1} &: Z \rightarrow \mathbf{2}^{X \times Y} \\ f^{-1} &: \{(x, y) \mid f(x, y) = z\} \end{aligned}$$

Two-dimensional histogram is defined by functions f and g :

$$f : X \times Y \rightarrow Z$$

$$g : X \times Y \rightarrow V$$

$$f^{-1} : Z \rightarrow \mathbf{2}^{X \times Y}$$

$$g^{-1} : V \rightarrow \mathbf{2}^{X \times Y}$$

$$f^{-1} : \{(x, y) | f(x, y) = z\}$$

$$g^{-1} : \{(x, y) | g(x, y) = v\}$$

There are many 2D histograms! One of the most useful is coocurrence *matrix*

$$M_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 3 \end{bmatrix}; \quad M_2 = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix};$$

$$z = [0 \ 1 \ 2 \ 3];$$
$$H_1(z) = [7 \ 5 \ 3 \ 1];$$

$$z = [0 \ 1 \ 2 \ 3];$$
$$H_2(z) = [7 \ 5 \ 3 \ 1];$$

Co-occurrence matrix relies on the condition (relation) between functions f and g (pixels), i.e.:

$$r = \{((x, y), (x, y + 1))\};$$

$$C_r = H_{fg}(z, v);$$

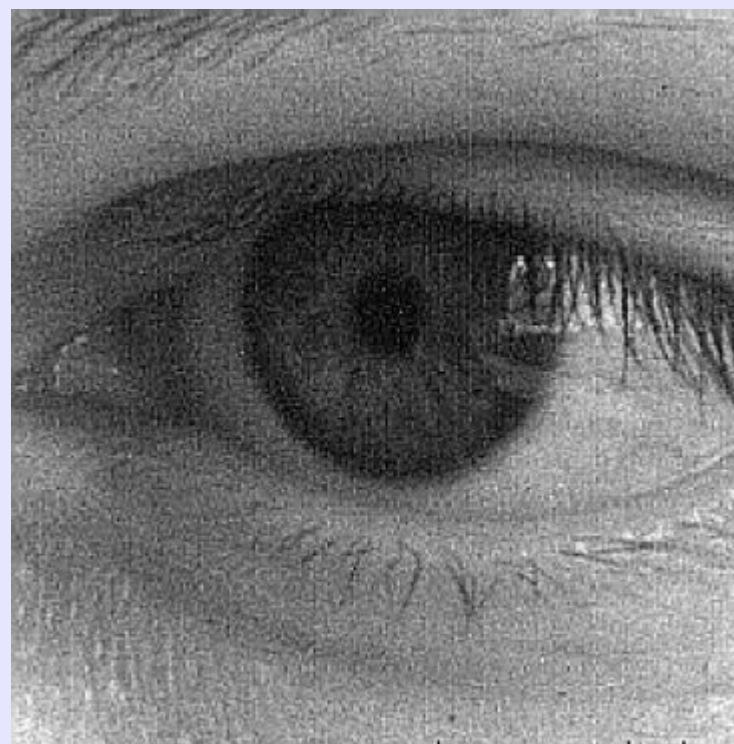
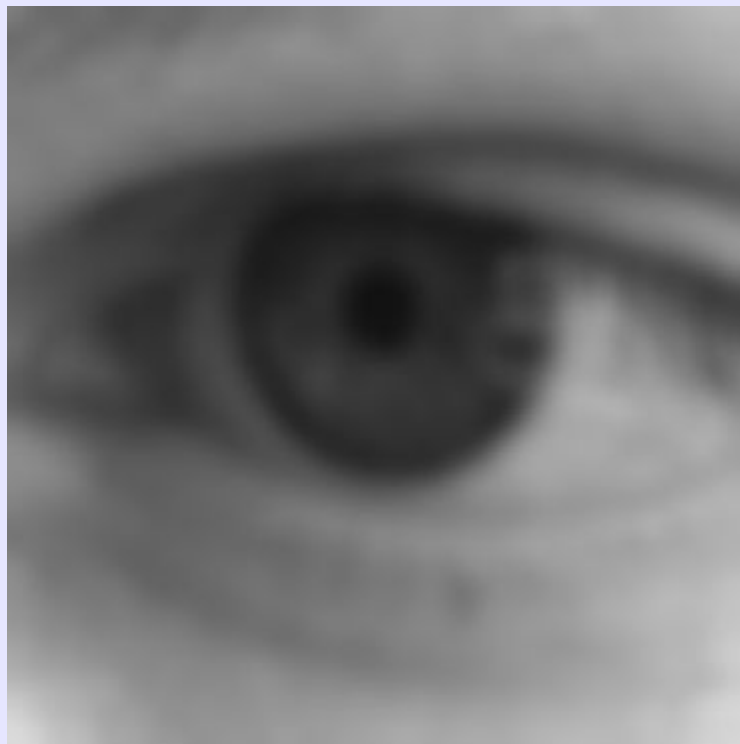
$$f(x, y) = g(x, y + 1);$$

$$C_{r_1} = \begin{bmatrix} 3 & 3 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$C_{r_2} = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix};$$



Example of calculation on real image – it helps when we want to tell if the image is crisp or blurred



Example of calculation on real image – it helps when we want to tell if the image is crisp or blurred

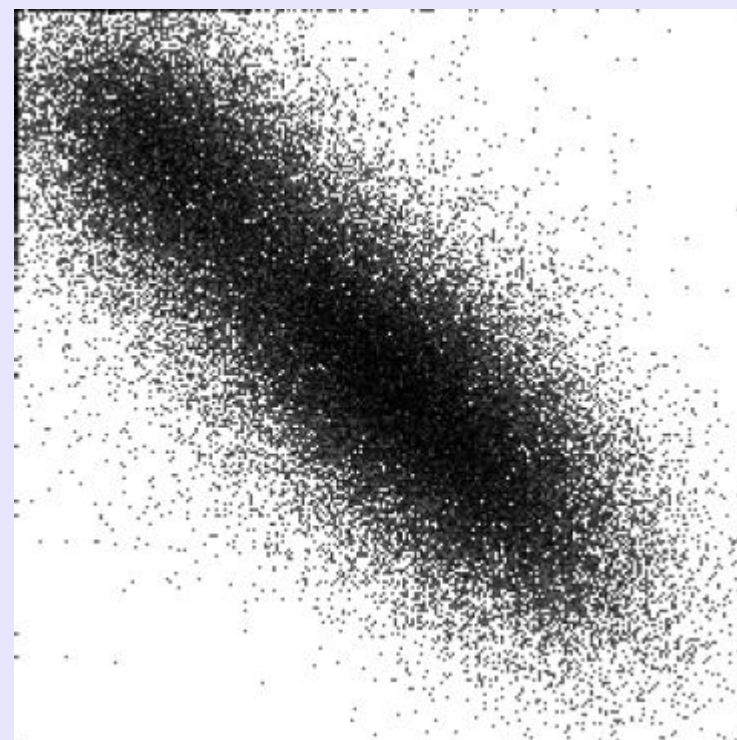
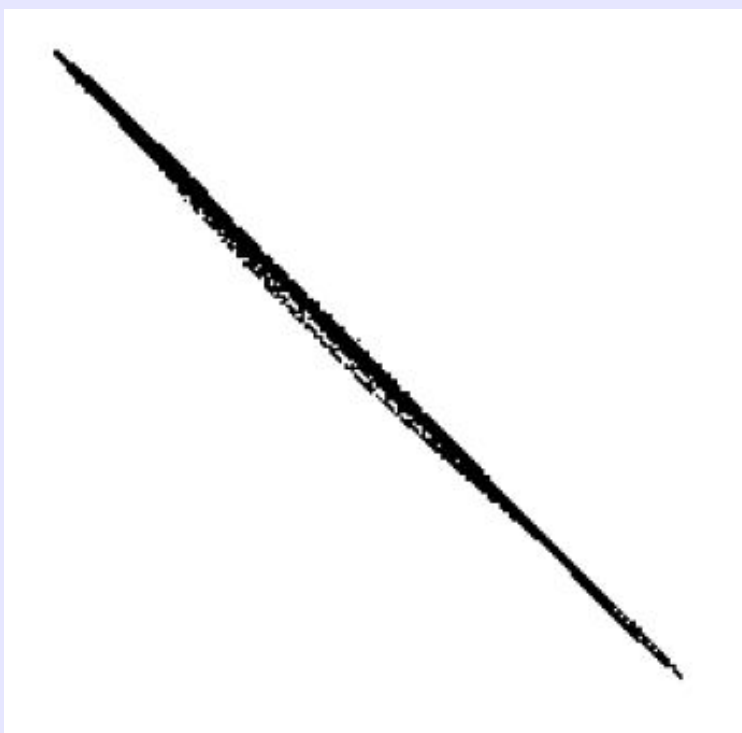




Image segmentation is the process of partitioning a digital image into **multiple segments** (sets of pixels, also known as superpixels).

The goal of segmentation is to **simplify** and/or change the representation of an image into something that is more meaningful and easier to analyze.

It is typically used to locate objects and boundaries (lines, curves, etc.) in images.

It is the process of assigning a label to every pixel in an image such that pixels with the same label share certain visual characteristics.



Point-based methods

- thresholding,
- clusterization (grouping)

Edge-based methods

- based on edge detection

Area-based methods

- region growing,
- region merging,
- region splitting,
- split & merge,
- watershed segmentation,

Hybrid methods

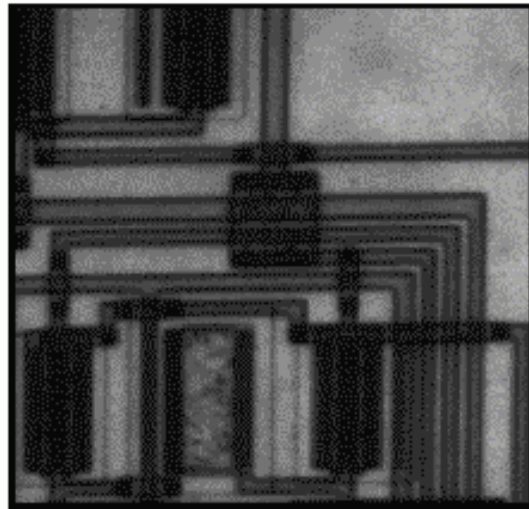
- using more than single approach, i.e. region growing with edge detection

Thresholding is the simplest method of image segmentation.

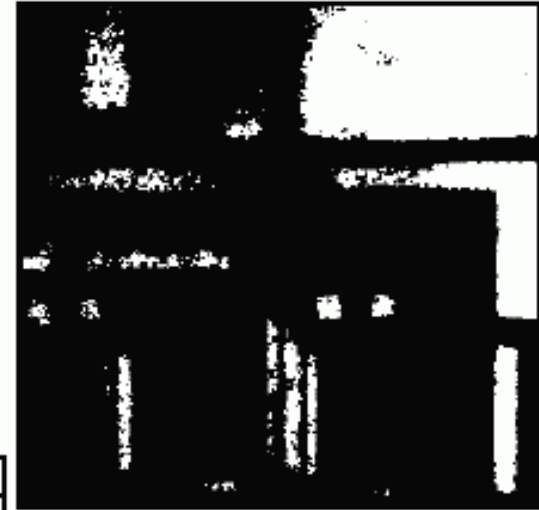
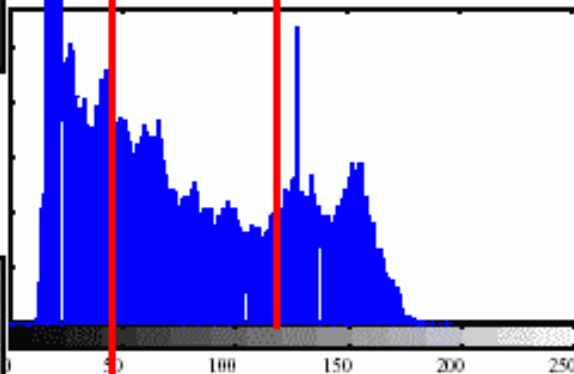
From a grayscale image, thresholding can be used to create binary images

T is the threshold.

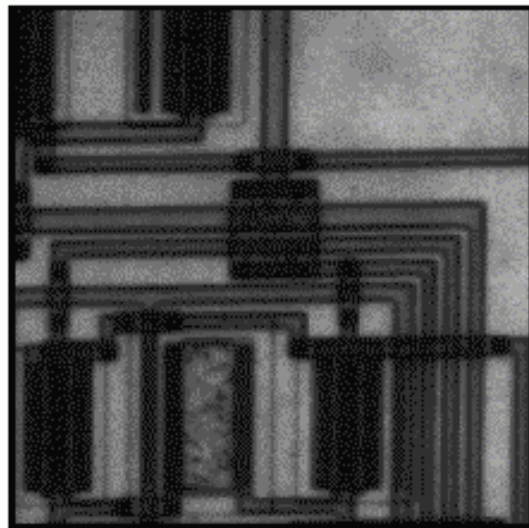
$$g(x, y) = \begin{cases} 1 & \text{for } f(x, y) \geq T \\ 0 & \text{for } f(x, y) < T \end{cases}$$



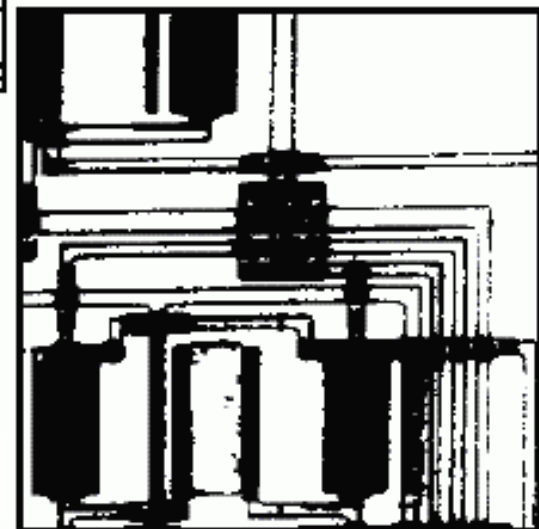
T=128



$$g(x, y) = \begin{cases} 256 & \text{for } f(x, y) \geq T \\ 0 & \text{for } f(x, y) < T \end{cases}$$



T=50



Intensity thresholding

We define m thresholds, i.e. $m=4$, $\{T_1, T_2, T_3, T_4\}$

After that we get 5 level image:

$$g(x, y) = \begin{cases} 0 & \text{for } f(x, y) \leq T_1 \\ 1 & \text{for } T_1 < f(x, y) \leq T_2 \\ 2 & \text{for } T_2 < f(x, y) \leq T_3 \\ 3 & \text{for } T_3 < f(x, y) \leq T_4 \\ 4 & \text{for } f(x, y) > T_4 \end{cases}$$

It changes only a limited group of pixels:

$$g(x, y) = \begin{cases} 0 & \text{for } f(x, y) \leq T \\ f(x, y) & \text{for } f(x, y) > T \end{cases}$$